

# Unified $V_{dd}$ - $V_{th}$ optimization based DVFM controller for a Logic block

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## Abstract

*In this paper analytical expressions for optimal  $V_{dd}$  and  $V_{th}$  to minimize energy for a given speed constraint are derived. These expressions are based on the EKV model for transistors and are valid in both strong inversion and sub threshold regions. The effect of gate leakage on the optimal  $V_{dd}$  and  $V_{th}$  is analyzed. A new gradient based algorithm for controlling  $V_{dd}$  and  $V_{th}$  based on delay and power monitoring results is proposed. A  $V_{dd}$ - $V_{th}$  controller which uses the algorithm to dynamically control the supply and threshold voltage of a representative logic block (Sum of Absolute Difference computation of an MPEG Decoder) is designed. Simulation results using 65 nm predictive Technology models are given.*

## 1. Introduction

Dynamically varying frequency of a logic block based on load requirements is effective in minimizing the energy consumptions. It has been shown that optimal supply and threshold voltages exist which minimizes the power consumed while meeting the speed constraint [1] [2] [3]. Most of the earlier attempts to derive the optimal  $V_{dd}$ ,  $V_{th}$  focus either on the strong inversion region or the sub threshold region of operation. But in a generic case where the frequency specifications of a circuit has a wide dynamic range operations in both regions may be necessary. In this paper delay and power models based on the EKV model [4] for the transistors are developed from which we obtain the optimal  $V_{dd}$  and  $V_{th}$  solutions which are valid in both the regions.

In [5] delay and power monitoring schemes for dynamically controlling  $V_{dd}$ ,  $V_{th}$  is proposed. The algorithm used in [5] varied  $V_{dd}$  based on the delay monitoring results and  $V_{th}$  based on power monitoring results. This is not satisfactory as the delay depends on  $V_{th}$  also and power has a  $V_{dd}$  dependency also. A control algorithm which varies  $V_{dd}$  and  $V_{th}$  in a combined fashion based on the delay and power will give a faster convergence and more accurate results. So a combined  $V_{dd}$ - $V_{th}$  control algorithm based on

the sensitivities of delay and power to  $V_{dd}$  and  $V_{th}$  is discussed in the paper. These sensitivities are obtained from the unified models developed.

The paper is organized as follows. Section 2 describes the analytical solution for optimal  $V_{dd}$ ,  $V_{th}$  based on the EKV model. Section 3 describes the proposed  $V_{dd}$ - $V_{th}$  control algorithm, delay and power monitoring circuits and the DVFM controller simulation results. Section 4 concludes the paper.

## 2 Unified $V_{dd}$ - $V_{th}$ optimization based on EKV model

### 2.1 Proposed EKV based current model

The  $V_{dd}$ - $V_{th}$  optimization for a given target frequency has been done in [1] for above threshold operation and in [2] [3] for sub threshold operation. The transistor current models in these work were based on the classical alpha power law model. Fig.1 shows the comparison of the SPICE simulated  $I_d$  values and the alpha power law model fitted values. There are two issues in using this model for  $V_{dd}$ - $V_{th}$  optimization.

- (i) It uses two different equations to model the current. A power law model for strong inversion region and an exponential model for subthreshold region. So  $V_{dd}$ - $V_{th}$  optimizations has to be done separately for these two regions.
- (ii) The model does not fit well for regions close to  $V_{th}$ .

To overcome these difficulties we chose to use a transistor current model based on the EKV model. The proposed model is given by

$$I_d = I_0 \left[ \ln \left( 1 + e^{\frac{V_{gs} - V_{th}}{\alpha N_s}} \right) \right]^\alpha \quad (1)$$

For high  $V_{gs}$  and low  $V_{gs}$  the proposed model reduces to the alpha power law model. But for  $V_{gs}$  values close to  $V_{th}$  it uses an interpolation function to predict the transistor current. This single equation closely follows the measured

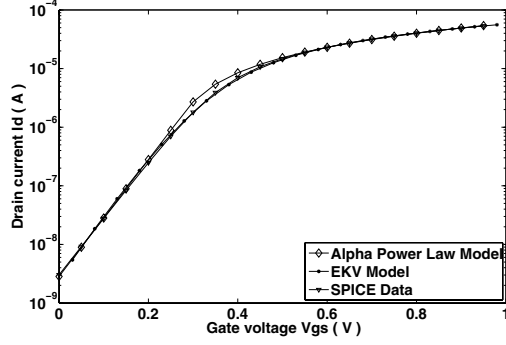


Figure 1.  $I_d - V_{gs}$  plot for 65nm transistor of  $w/l = 65\text{nm}/65\text{nm}$  at  $V_{ds} = 1\text{V}$ .

values in all regions with an maximum error of around 15%. Fig.1 shows the comparison between the spice simulated transistor currents and that given by (1).

## 2.2 Test circuit

To evaluate the validity of the proposed models a test circuit given in [8] is used. The circuit has a ring oscillator made of 11 NAND gates feeding chains of NAND gates. All the SPICE results given in this section have been obtained by the simulations of this test circuit.

## 2.3 Delay model

A simple delay model for a gate is given by

$$t_{gate} = \frac{KC_L V_{dd} \theta}{I_0 [\ln(1 + e^{\frac{V_{dd} - V_{th0max} + \eta V_{dd}}{\alpha N_s}})]^\alpha} \quad (2)$$

where  $N_s$  is the subthreshold slope,  $V_{thmax}$  is the maximum  $V_{th}$  across temperature and process corners,  $C_L$  is the load of the gate and  $K$  is a delay fitting parameter.  $\theta$  is a degradation factor to account for increase in delay due to finite rise/fall times.  $\theta$  is a function of  $V_{dd}$ ,  $V_{th}$ . Through simulations the degradation due to slope was found and modelled as a function  $V_{dd}$ ,  $V_{th}$  as

$$\theta = \frac{1}{[\ln(1 + e^{\frac{V_{dd} - V_{th0max} + \eta V_{dd}}{\alpha N_s}})]^\beta} \quad (3)$$

$\beta$  is a fitting parameter. Fig.2 shows the accuracy of the model in predicting the delay across a wide range of  $V_{dd}$  for 65nm (PTM) [9] and 90nm (UMC) processes. Maximum discrepancies of around 17% were seen across a wide range of  $V_{dd}$  and  $V_{th}$ .

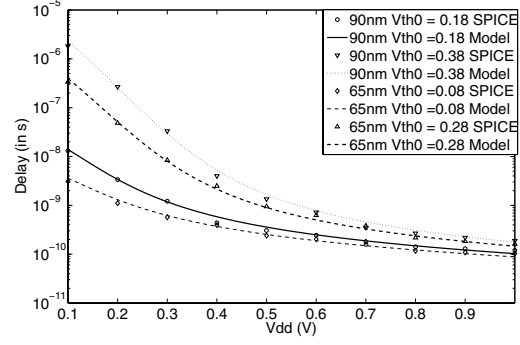


Figure 2. Comparisons of proposed model and SPICE data for 90nm and 65nm processes. Delays shown are that of a ring oscillator in the test circuit.

## 2.4 Power model

Dynamic power is given by

$$P_{dyn} = \alpha C_{Leff} V_{dd}^2 f \quad (4)$$

Where  $C_{Leff}$  is the effective switching capacitance and  $\alpha$  is the activity factor. In this work the contribution of short circuit power dissipation has been neglected for simplicity as it is a small fraction of total power [1]. Leakage current in deep submicron devices is predominantly due to subthreshold leakage and gate leakage. In this optimization the gate leakage current is neglected. However in a subsequent section the validity of this assumption and its limitations are discussed. Fig.3 shows the comparison of leakage current of the test circuit using 65nm PTM process with the corresponding models.

$$I_{leak} = I_0 W_{eff} e^{-\frac{(V_{th0min} - \eta V_{dd})}{N_s}} \quad (5)$$

$$P_{leak} = I_0 W_{eff} e^{-\frac{(V_{th0min} - \eta V_{dd})}{N_s}} V_{dd} \quad (6)$$

$$V_{th0min} = V_{th0max} - \Delta V_{th0} \quad (7)$$

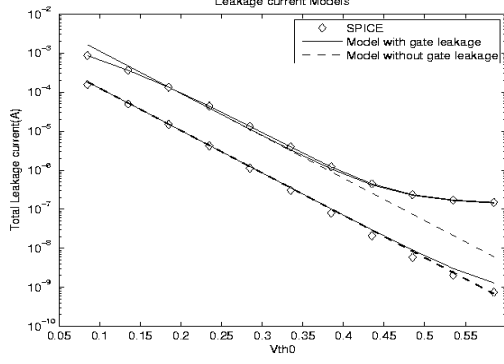
$\Delta V_{th0}$  is peak-to-peak  $V_{th0}$  variation across process corners and temperature.  $V_{th0min}$  is lowest  $V_{th0}$  in operation temperature and process variation range [1]. It is to be noted that delay is calculated at  $V_{th0max}$  and leakage power at  $V_{th0min}$  as in [1]. The total power is given by

$$P_t = P_{leak} + P_{dyn} \quad (8)$$

## 2.5 $V_{dd} - V_{th}$ optimization

Using above models optimization for minimum energy is done. solving (2) for  $V_{thmin}$  in terms of  $V_{dd}$

$$V_{th0min} = (1 + \eta) V_{dd} - \alpha N_s \ln(e^{\frac{1}{\alpha N_s}} - 1) - \Delta V_{th0} \quad (9)$$



**Figure 3. Leakage current of the test circuit. Comparison of SPICE results, equation(5) and equation(14).**

Where  $\gamma = \alpha + \beta$ . Using (6), (9) leakage power is given by

$$P_{leak} = I_0 W_{eff} e^{-\frac{(1+\eta)V_{dd} - \Delta V_{th0}}{N_s}} (e^{\chi V_{dd}^{\frac{1}{\gamma}}} - 1)^{\alpha} V_{dd} \quad (10)$$

where

$$\chi = \left( \frac{K C_L f L_d}{I_0} \right)^{\frac{1}{\gamma}} \quad (11)$$

the optimum values are given by solving

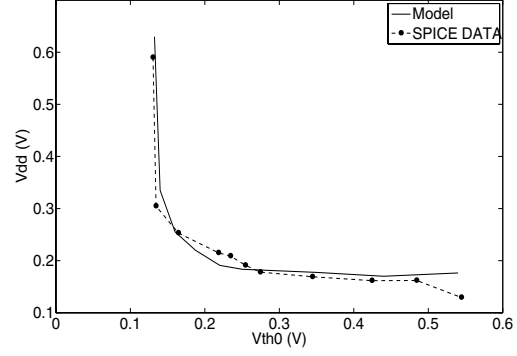
$$\frac{dP_t}{dV_{dd}} = 2\alpha C_{Leff} V_{dd} f + P_{leak} \left\{ \frac{1}{V_{dd}} - \frac{1+\eta}{N_s} + \frac{\alpha \chi V_{dd}^{\frac{1}{\gamma}-1} e^{\chi V_{dd}^{\frac{1}{\gamma}}}}{\gamma (e^{\chi V_{dd}^{\frac{1}{\gamma}}} - 1)} \right\} = 0 \quad (12)$$

$V_{ddopt}$  is obtained by solving (12) for  $V_{dd}$ . Equation (12) is transcendental and is solved numerically.  $V_{thopt}$  obtained by substituting  $V_{ddopt}$  in (9).

Fig.4 compares the optimal  $V_{dd}/V_{th}$  values obtained using the models with those obtained from exhaustive SPICE simulation. The model and SPICE values are very close, further validating the models.

## 2.6 Discussion on the optimization results

- (i) In the strong inversion region  $V_{thopt}$  remains almost constant with frequency whereas  $V_{dd}$  varies to achieve the necessary speed whereas in the subthreshold region the  $V_{ddopt}$  remains almost constant and  $V_{thopt}$  varies with frequency [3] [2] [1].
- (ii) The optimal energy points occur where the leakage energy and the dynamic energy are comparable. [1] report that optimal energy point occurs when the ratio of



**Figure 4. Optimal energy points for various frequencies on the  $V_{dd}$ - $V_{th}$  plane for 65nm PTM process. Model and SPICE Data comparison.**

dynamic to leakage power remains constant. This is true both in the strong inversion and the sub threshold regions but the optimality occurs for different ratios in these two regions. If a single ratio is used for the whole range of frequencies, the power level for some range of frequencies will be sub optimal. In fig.5 ratio of energy along the optimal curve to that along the curve  $E_{dyn}/E_{leak} = \text{constant}$  is plotted for different delays. The sub optimality is within 10% as shown in fig.5.

## 2.7 Effect of gate leakage on the optimal curve

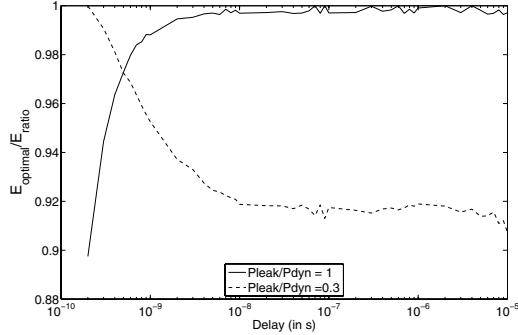
We model Gate leakage of a circuit empirically as

$$I_{gateleak} = I_{0gate} * V_{dd}^{k1} * e^{-k2 * V_{th0min}} \quad (13)$$

$$I_{leak} = I_{gateleak} + I_{subleak} \quad (14)$$

Where  $k1, k2$  are fitting parameters.

Fig.3 shows the accuracy of the empirical fit for 65nm process. Gate leakage is strongly dependent on  $V_{dd}$  and increases with  $V_{dd}$ . However in the strong inversion region, optimal  $V_{th}$  is very low and hence subthreshold leakage dominates. In the sub-threshold regions  $V_{dd}$  is very low and hence gate leakage is again negligible. Hence gate leakage term affects the optimal curve very little. When the frequency is extremely low and optimal  $V_{th0}$  is very high the subthreshold leakage falls even below the gate leakage. Only for such extremely low frequencies gate leakage tends to affect the optimal  $V_{dd} - V_{th}$  curve.



**Figure 5. Suboptimality in choosing constant Eleak/Edyn ratio as the minimum energy criteria.**

### 3 DVFM controller

A DVFM controller based on delay and power measurement results is shown in fig.6(a). It adjusts the supply and body bias to achieve minimum power for a frequency target. The control algorithm uses the measured values of delay and power to adjust  $V_{dd}$  and body bias and is described next.

#### 3.1 Gradient based $V_{dd} - V_{th}$ control algorithm

In [5]  $V_{dd}$  is controlled based on the results of the delay measurement and  $V_{th}$  is varied based on the results of power measurements.  $V_{dd}$  and  $V_{th}$  affects both delay and power and hence a combined control of  $V_{dd}$  and  $V_{th}$  based on the power and delay measurements will be more effective. We propose a new control algorithm below.

The difference between targeted delay( $D_{target}$ ) and the actual circuit delay( $D_{actual}$ ) is obtained. Also the ratio of Leakage Energy to Dynamic Energy( $Eratio$ ) is obtained and its difference from the optimal Energy ratio ( $Eratio_{optimal}$ ) is computed.

$$delD = D_{target} - D_{actual} \quad (15)$$

$$delEratio = Eratio_{target} - Eratio_{actual} \quad (16)$$

The Sensitivity matrix is defined as

$$S = \begin{bmatrix} \frac{\partial D}{\partial V_{dd}} & \frac{\partial D}{\partial V_{th}} \\ \frac{\partial Eratio}{\partial V_{dd}} & \frac{\partial Eratio}{\partial V_{th}} \end{bmatrix} \quad (17)$$

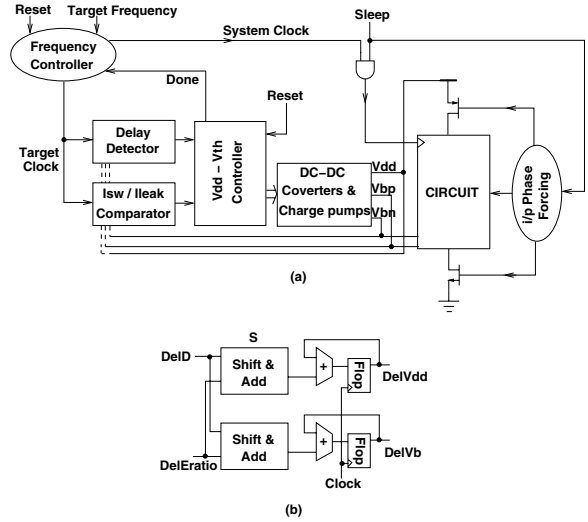
We can write

$$[delD \ delEratio]^T = S * [delvdd \ delvth]^T \quad (18)$$

Then the direction in which we should move in the  $V_{dd} - V_{th}$  space to reach the optimal point is computed by inverting the above equation.

$$[delvdd \ delvth]^T = R * [delD \ delEratio]^T \quad (19)$$

where R is related to the inverse of the sensitivity matrix S.

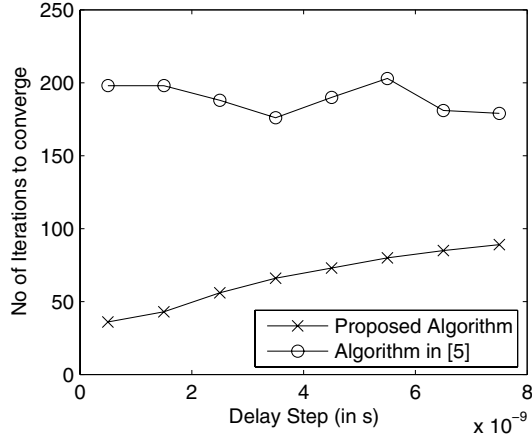


**Figure 6. (a)Block Diagram of DVFM controller. (b)Proposed  $V_{dd} - V_{th}$  controller.**

The matrix multiplication in (19) is implemented in a shift and add fashion as shown in fig.6(b). Choosing the elements of R is critical as they control the dynamics and stability of the loop. The following heuristic steps are developed to avoid oscillations in the proposed algorithm.

- (i) Given a range of frequencies the optimal  $V_{dd} - V_{th}$  points are estimated using the delay and power models given in equations (2) and (8). The range of  $V_{dd}$  and  $V_{th}$  is thus known.
- (ii) The sensitivity matrix S is evaluated using (17) at each point in this operational range. Then  $S^{-1}$  is computed.  $R(i,j)$  is then computed as  $\min S^{-1}(i,j)$  over the operational range.

The performance of the proposed algorithm was evaluated using MATLAB simulations. The operation of the algorithm to control the  $V_{dd}$  and  $V_{th}$  of test circuit for a frequency range on 100M - 1G was simulated. The initial values of  $V_{dd}$  and  $V_{th}$  was chosen to be the optimal  $V_{dd} - V_{th}$  for a frequency of 2GHz. A step decrease in frequency is then given and the number of iterations to converge was found for the proposed algorithm and the algorithm in [5]. The proposed algorithm is faster by more than 50% than the previous work as shown in fig.7. In a DVFM



**Figure 7. Performance comparison of the proposed algorithm and algorithm in [5].**

controller each iteration must be followed by a change in control to the power controller (to change  $V_{dd}$  and  $V_{th}$ ). In algorithm proposed in [5] in each iteration either  $V_{dd}$  or  $V_{th}$  is changed. So the convergence time is

$$T_c = \text{No. of } V_{dd} \text{ iterations} \times \text{time taken per } V_{dd} \text{ step} + \text{No. of } V_{th} \text{ iterations} \times \text{time taken per } V_{th} \text{ step}$$

In the proposed algorithm  $V_{dd}$  and  $V_{th}$  are changed simultaneously. So the convergence time is

$$T_c = \text{No. of iterations} \times \text{Max}(\text{time taken per } V_{dd} \text{ step, time taken per } V_{th} \text{ step})$$

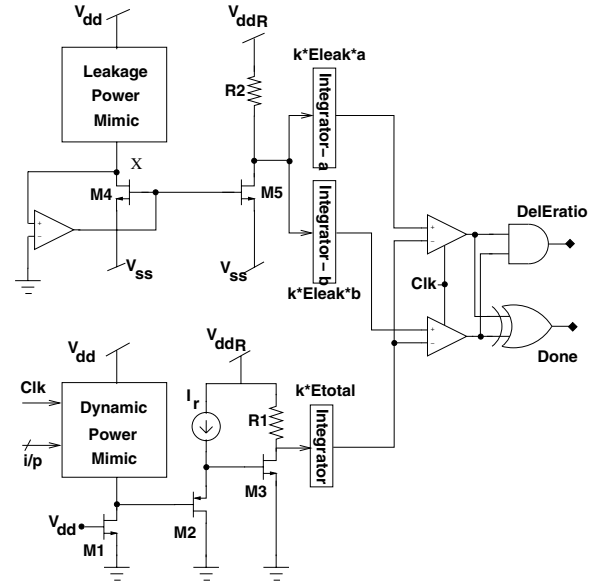
Time taken per  $V_{dd}$  and  $V_{th}$  step depends on the response time of the power regulators, it can be in the order of microseconds. Hence in [5] algorithm even if equal no. of  $V_{dd}$  and  $V_{th}$  steps are assumed, the proposed algorithm performs better.

### 3.2 Architecture of the Logic block

The logic block to compute the sum of sixteen 16 bit numbers is similar to the SAD computation architecture used in [6]. It uses 6 stages of carry save adders and the final carry propagation is done using a Hans Carlson adder. A pipelining of 12 FO4 delay per stage found to be optimal [7] is used.

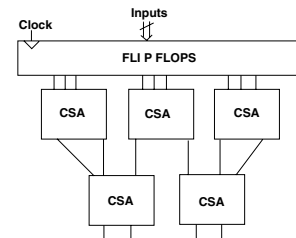
### 3.3 Delay and Power monitoring Circuits

The Delay monitoring circuit given in [5] is used. The monitoring circuit to measure the total energy consumed by the circuit is given in fig.8. The SAD computing structure is highly regular and is composed of adders, flops and wires only. Due to the regularity of the structure we believe that the power mimic made of adders, flops and wires in a proportionate ratio should mimic the total circuit power across



**Figure 8.  $I_{leak} - I_{sw}$  comparator.**

wide range of  $V_{dd}$  and  $V_{th}$  satisfactorily. The mimic used is shown in fig.9. The voltage drop across the sleep transistor is proportional to the current flowing through it and hence can serve as a measure of the instantaneous current flow. The total energy measurement is done by switching the inputs of the power mimic using a counter and integrating the drop across the sleep transistor over a few cycles. Since the voltage across the sleep transistor is close to ground a PMOS level shifter is used to shift the voltage up by  $V_{th}$ . The shifted voltage is amplified and integrated to obtain a voltage proportional to the total energy consumed. The leakage power measurement circuit proposed in [5] is used. The Op-Amp shown maintains the node x at ground potential. The leakage current of the mimic flows through M4 which is mirrored to M5 producing a proportional voltage across R2. This voltage is then integrated and compared with the total energy using a clocked comparator to generate *delEratio*. A small window for comparison is provided with scaling factors a and b. The circuit proposed uses a negative voltage  $V_{ss}$  of -0.75V. The linearity of the total energy and leakage energy are shown in fig.10(a) and fig.10(b).



**Figure 9. Power Mimic.**

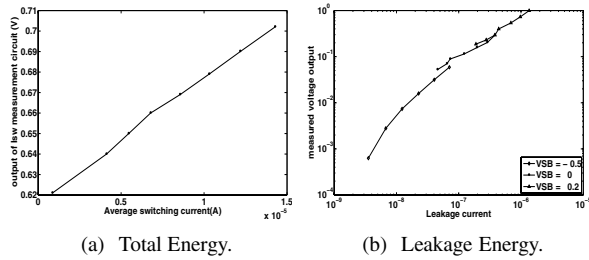


Figure 10. Linearity of Measurements

### 3.4 DVFM Controller Simulation results

The DVFM controller with the measurement circuits was simulated with behavioral models for all the digital components of the controller. 65nm PTM model is used to build the logic block and the measurement circuits. An ideal DC-DC converter with voltage resolution of around 10mV was assumed in the behavioral model. The target frequencies Vs the optimal  $V_{dd}$ - $V_{th}$  points are given in fig.11(a) and the power consumed by the block at different frequencies is given in fig.11(b). In the super threshold region the optimal  $V_{th}$  turns out to be lesser than the  $V_{th}$  obtainable by forward body bias. So the controller settles down the minimum obtainable  $V_{th}$  and  $V_{dd}$  to maintain the delay constraint. That is the controller settles to the reachable optimal point in the  $V_{dd}$ - $V_{th}$  plane. The power consumed by the circuit for various frequencies at the optimal  $V_{dd}$ - $V_{th}$  given by the controller is given in fig.11(b). The leakage power and the dynamic power are comparable indicating that the optimal points given by the controller are indeed close to the minimal power points.

## 4 Conclusions

Analytical expressions for optimal  $V_{dd}$ - $V_{th}$  valid in both strong inversion and sub threshold regions has been derived. Gate leakage is found to affect the optimal  $V_{dd}$ - $V_{th}$  curve very little except for very low frequencies. Minimum energy operation for a given frequency occurs when the leakage energy and the dynamic energy are comparable to each other and this criteria holds quite well for a wide range of frequencies.  $V_{dd}$ - $V_{th}$  controller based on a new control algorithm has been designed and simulated using behavioral models. The proposed algorithm performs better than the previously reported algorithms in terms of convergence speed.

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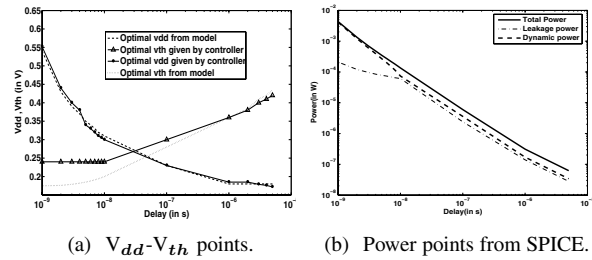


Figure 11. Optimal Results.

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